Contour dynamics for 2D active scalars

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Abstract:

In this talk we discuss two free boundary problems given by fluid domains which are weak solutions of incompressible equations. We consider the contour dynamics Muskat problem and the evolution of a sharp front by the 2D surface Quasi-geostrophic equation. Both systems are described by means of a transport equation for the active scalar $\rho(x,t)$ which takes constant values on complementary domains. The velocity field is determined by $\rho(x,t)$ by singular integral operators. However the solutions of these two physical scenarios have completely different outcomes regarding well-posedness and regularity issues.

References:

1. G. Baker, D. Meiron and S. Orszag. Generalized vortex methods for free-surface flow problems. *J. Fluid Mech.* 123 477-501, 1982.
2. A. L. Bertozzi and P. Constantin. Global regularity for vortex patches. *Comm. Math. Phys.* 152 (1), 19-28, 1993.
3. J.Y. Chemin. Persistence of geometric structures in two-dimensional incompressible fluids. *Ann. Sci. Ecole. Norm. Sup.* 26 (4), 517-542, 1993.
4. P. Constantin, A. J. Majda, and E. Tabak. Formation of strong fronts in the 2-D quasigeostrophic thermal active scalar. *Nonlinearity*, 7, 1495-1533, 1994.
5. P. Constantin. Scaling exponents for active scalars. *Journal of Statistical Physics*, vol 90, 3-4, 571-595, 1998.
6. A. Córdoba, D. Córdoba and F. Gancedo. Interface evolution: the Hele-Shaw and Muskat problems. *To appear in Annals of Math.*
7. D. Córdoba, M. A. Fontelos, A. M. Mancho and J. L. Rodrigo. Evidence of singularities for a family of contour dynamics equations. *Proc. Natl. Acad. Sci. USA* 102, 5949-5952, 2005.
8. D. Córdoba and F. Gancedo. Contour dynamics of incompressible 3-D fluids in a porous medium with different densities. *Comm. Math. Phys.* 273, 2, 445-471 (2007).

9. Córdoba, F. Gancedo and R. Orive. A note on the interface dynamics for convection in porous media. *Physica D*, 237, 1488-1497, 2008.
10. F. Gancedo. Existence for the α-patch model and the QG sharp front in Sobolev spaces. *Adv. Math.*, 217, 2569-2598, 2008.
11. T.Y. Hou, J.S. Lowengrub and M.J. Shelley. The long-time motion of vortex sheets with surface tension. *Phys. Fluids*, 9, no. 7, 1933–1954, 1997.
12. M. Muskat. The flow of homogeneous fluids through porous media. *New York*, Springer 1982.
13. Lord Rayleigh (J.W. Strutt), On the instability of jets. *Proc. Lond. Math. Soc.* 10, 413, 1879.
14. J.L. Rodrigo. On the Evolution of Sharp Fronts for the Quasi-Geostrophic Equation. *Comm. Pure and Appl. Math.*, 58, 0821-0866, 2005.
15. P.G. Saffman and Taylor. The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid. *Proc. R. Soc. London, Ser. A* 245, 312-329, 1958.
16. M. Siegel, R. Caflisch and S. Howison. Global Existence, Singular Solutions, and Ill-Posedness for the Muskat Problem. *Comm. Pure and Appl. Math.*, 57, 1374-1411, 2004.
17. E. Stein. Singular Integrals and Differentiability Properties of Function. *Princeton University Press.* Princeton, NJ, 1970.